Optimal control of one and two Josephson phase qubit systems

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Abstract

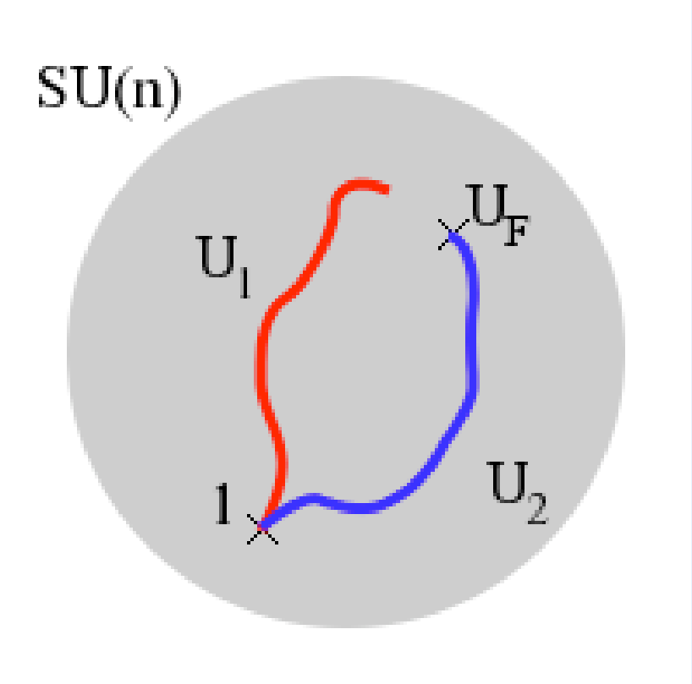
Josephson junction qubits are promising candidates for quantum computing. For the implementation of quantum algorithms one needs fast one and two-qubit gates with low error rates. We apply optimal control theory to a single Josephson phase qubit, described as a weakly nonlinear three-level system. By using the GRAPE algorithm developed by the Khaneja and Glaser groups [1], we find smooth, shaped pulses that significantly reduce the required time for high-fidelity unitary gates compared to simple rectangular pulses. Our pulses make extensive use of the third non-qubit level during application. We investigate optimal control of two capacitively coupled Josephson phase qubits. We find pulses that efficiently perform the two-qubit operations needed for quantum algorithms. For both the single and two-qubit system, we formulate performance indices which do not depend on the phases of non-qubit states. Progress towards experimentally realizing these pulses is also discussed.

The GRAPE algorithm

Problem

$$\dot{U}(t) = -i(H_{\mathrm{drift}} + \sum_k \lambda_k(t) H_k) U(t),\ U(0) = \mathbf{1}$$

Find λ_k to hit U_{F} at time T!



Hit given U_{F} in SU(n)

Performance index

Min. $||U_{\mathrm{F}} - U(T)||^2 \Leftrightarrow$ Max. $\phi = \mathrm{ReTr}(U_{\mathrm{F}}^\dagger U(T))$
If global phase irrelevant, maximize

$$\phi = |Tr(U_{\mathrm{F}}^\dagger U(T))|^2$$

Gradient

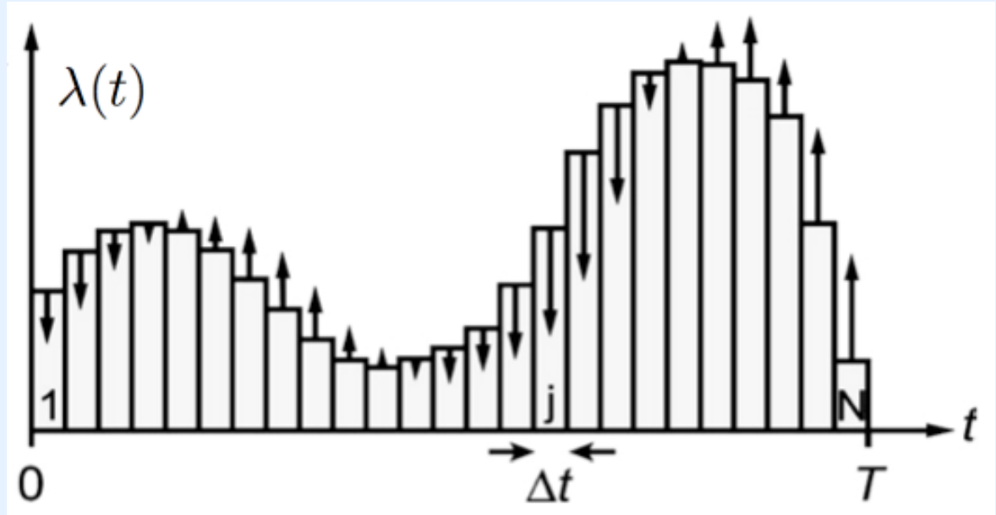
Gradient ascent procedure with analytical gradient

$$\frac{\partial \phi}{\partial \lambda_k(j)} = -2\Delta t \mathrm{Re}\left[\left(\mathrm{Tr} U_{\mathrm{F}}^\dagger U_N \dots U_{j+1} i H_k U_j \dots U_1\right) \left(\mathrm{Tr} U_1^\dagger \dots U_j^\dagger U_{j+1}^\dagger \dots U_N^\dagger U_{\mathrm{F}}\right)\right]$$

with trotterized time-step propagators

$$U_i = \exp\left(-i\Delta t (H_{\mathrm{drift}} + \sum_k \lambda_k(t_i) H_k)\right)$$

Only two full propagations per step needed!

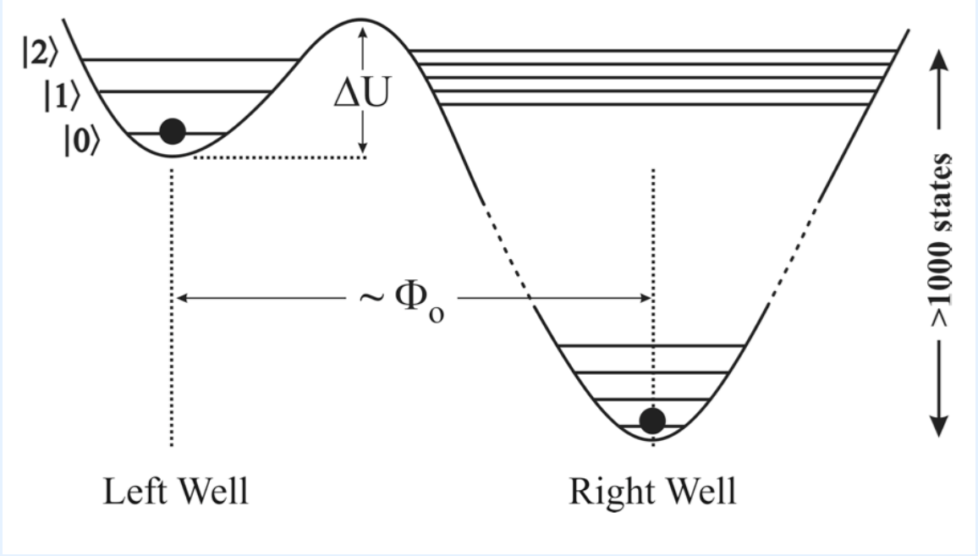


Example of pulse and gradient [1]

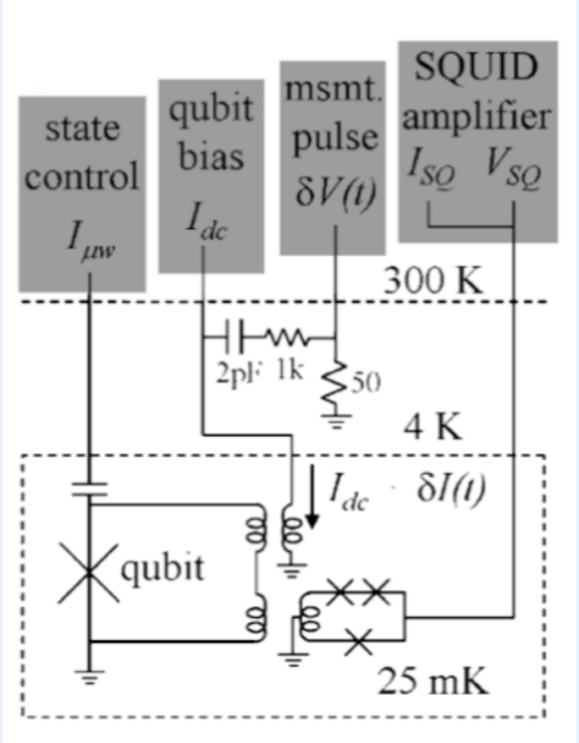
Josephson phase qubit

RWA Hamiltonian, ω_{10} resonant driving, $\Delta\omega = |\omega_{21} - \omega_{10}| \approx 0.1\omega_{10}$, control $\lambda(t)$ proportional to microwave pulse amplitude

$$H(t) = \begin{pmatrix} -\Delta\omega & \sqrt{2}\lambda(t) & 0 \\ \sqrt{2}\lambda(t) & 0 & \lambda(t) \\ 0 & \lambda(t) & 0 \end{pmatrix}$$



Potential for phase difference across flux-biased Josephson junction [2]



Phase qubit circuit realized at Martinis' group at UC Santa Barbara [2]

Optimal one qubit gates

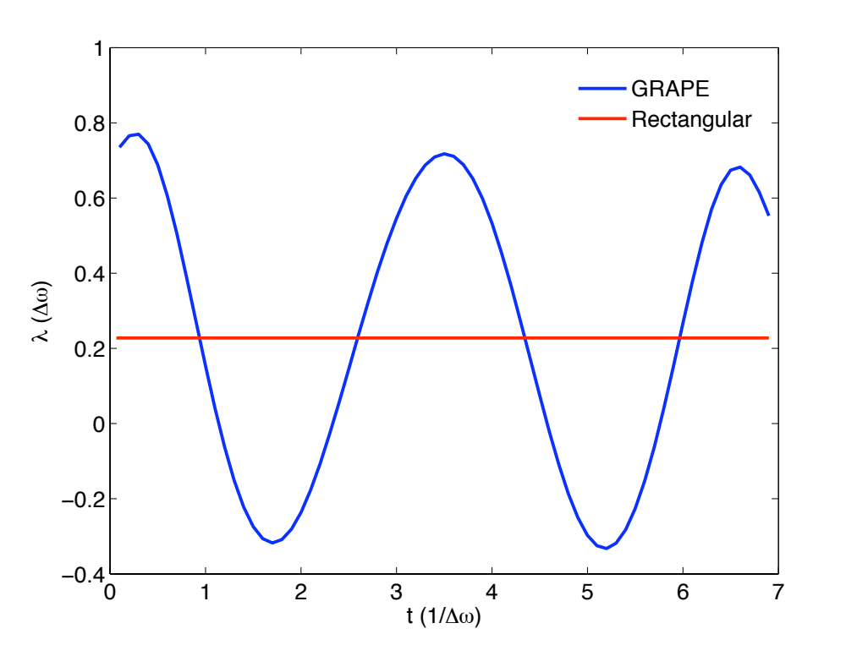
$$\phi = \frac{1}{4}|(U_{\mathrm{F}}^\dagger U(T))_{22} + (U_{\mathrm{F}}^\dagger U(T))_{33}|^2$$

$$\mathrm{NOT}:\ U_{\mathrm{F}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

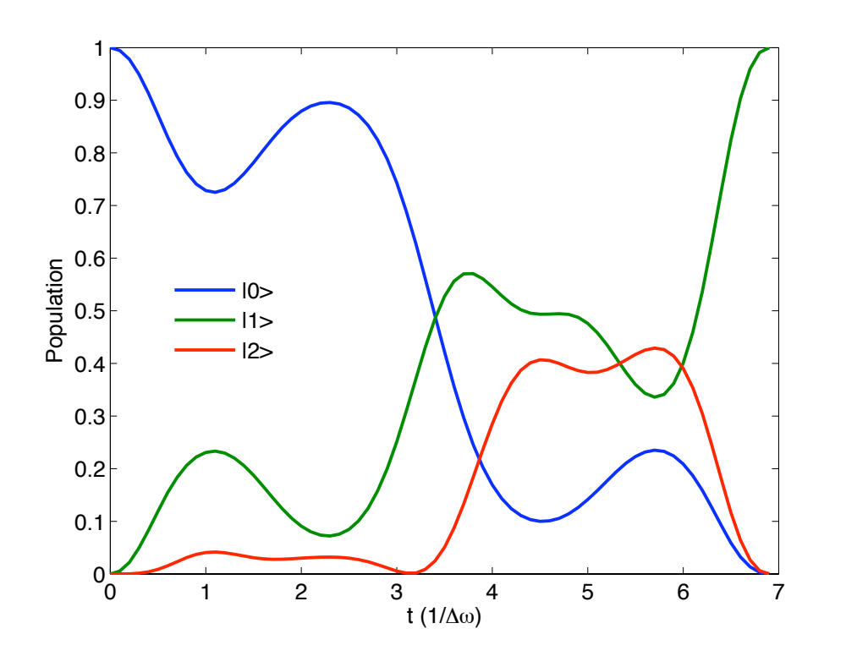
- Smooth pulses, but initial rise to sharp for experiment
- $(2n+1)\pi$ pulse on qubit, $2n\pi$ pulse on leakage
- Long rectangular pulses perform well (weak driving)
- Fault tolerant to first order

- Huge performance increase over rectangular Rabi pulse (error below 10^{-8} for $T > 6.5/\Delta\omega$)
- It makes sense to use ϕ, instead of

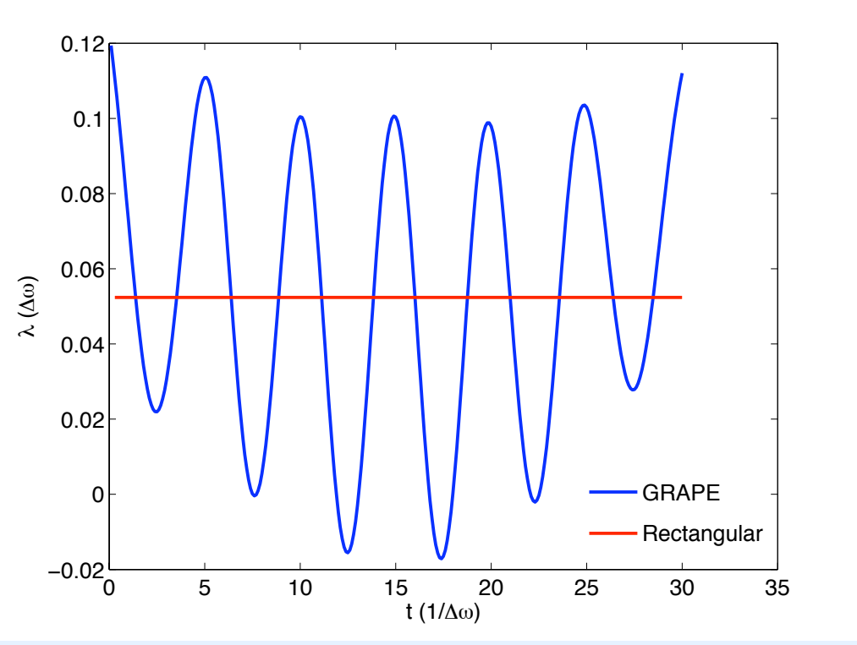
$$\phi_2 = \frac{1}{9}|Tr(U_F^\dagger U(T))|^2$$



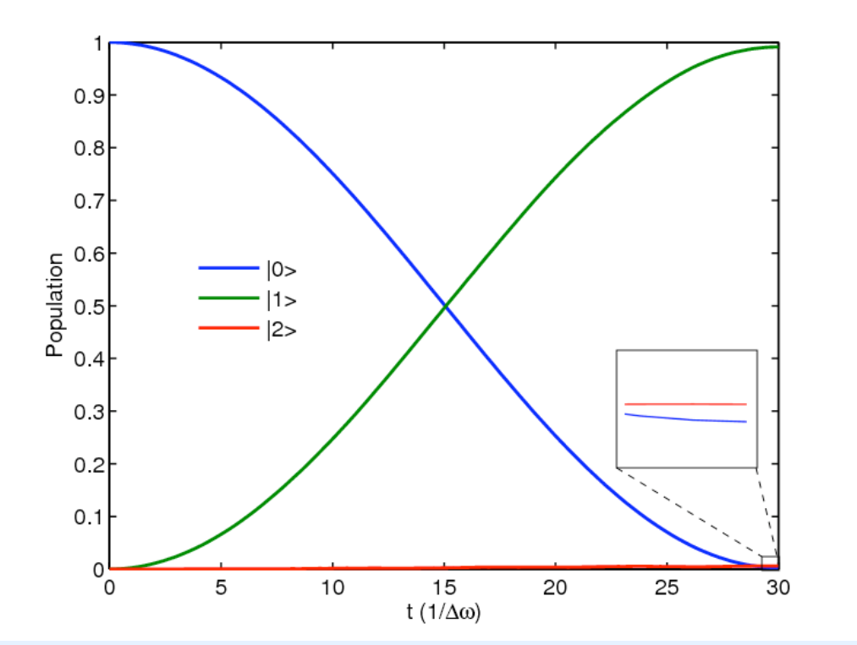
Short GRAPE pulse



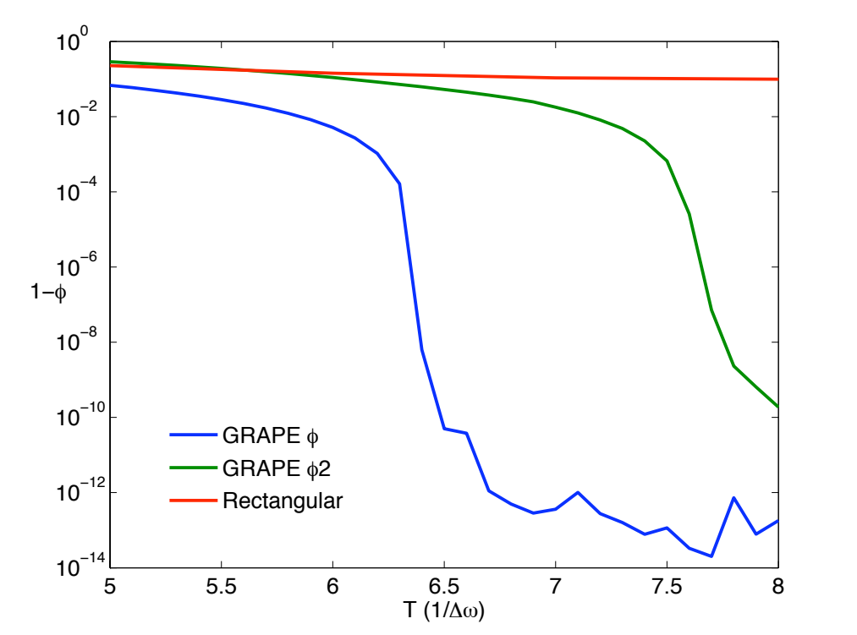
Population of GRAPE pulse



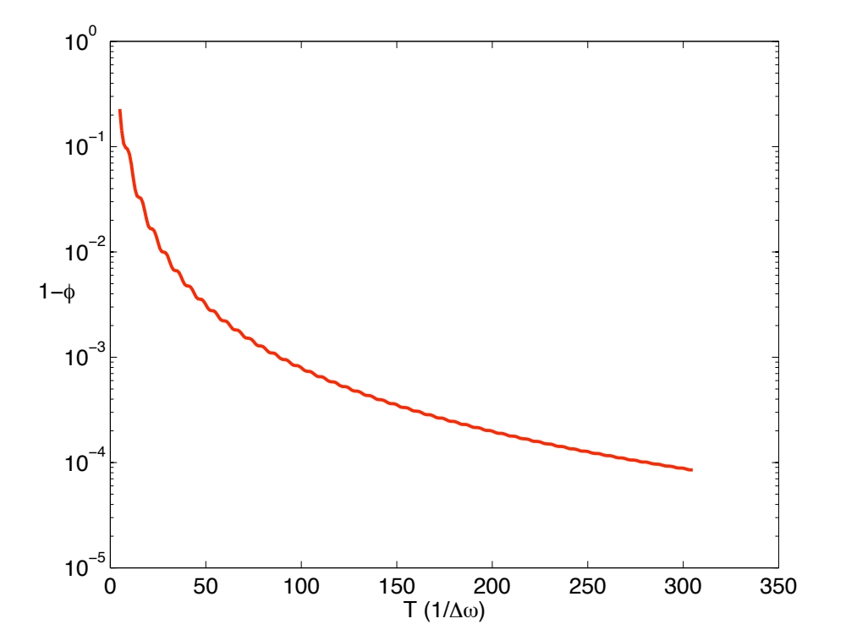
Long GRAPE pulse



Population of rectangular pulse



Error of GRAPE pulses versus total pulse time



Error of rectangular pulses versus total pulse time

Towards better pulse shapes

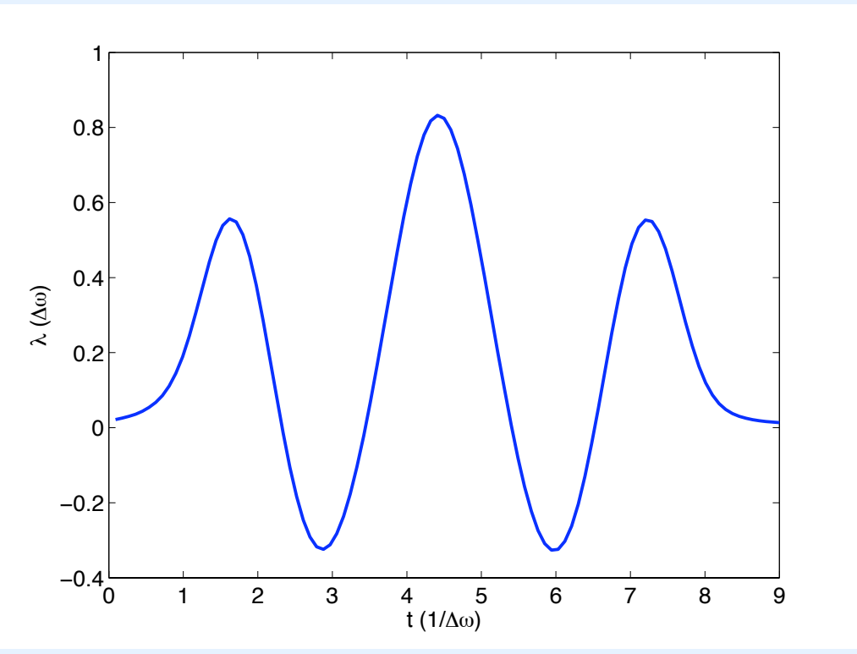
Penalize sharp rises (with suitably chosen penalty strength $\gamma(t)$)

$$\tilde{\phi} = \phi + \int_0^T \gamma(t)\lambda(t)^2 dt$$

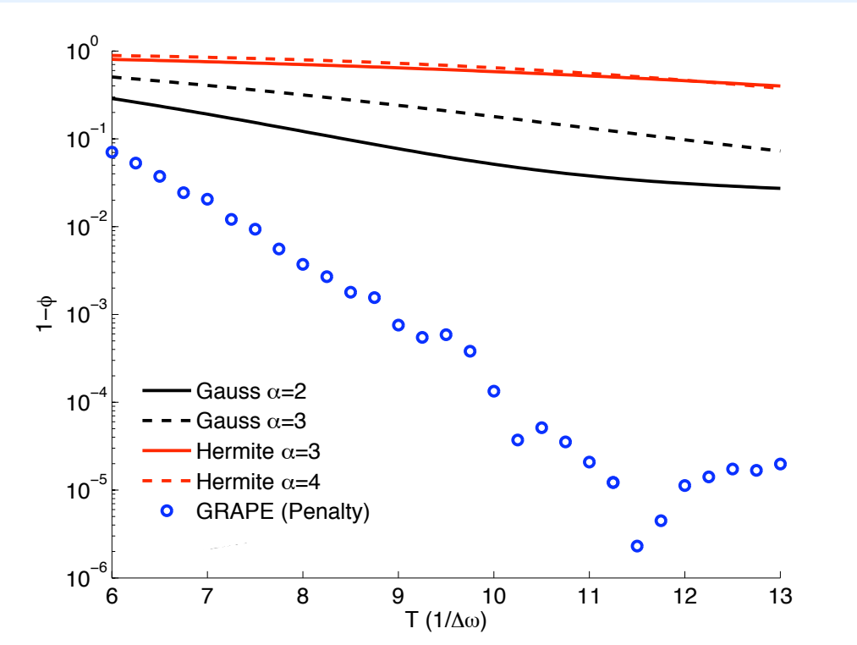
or Gaussian pulse shapes (currently used by Martinis' group [3])

$$\lambda_{\mathrm{G}}(t) = 2a\frac{\alpha}{T}\exp\left(-2\alpha^2\frac{(t-\frac{T}{2})^2}{T^2}\right)$$

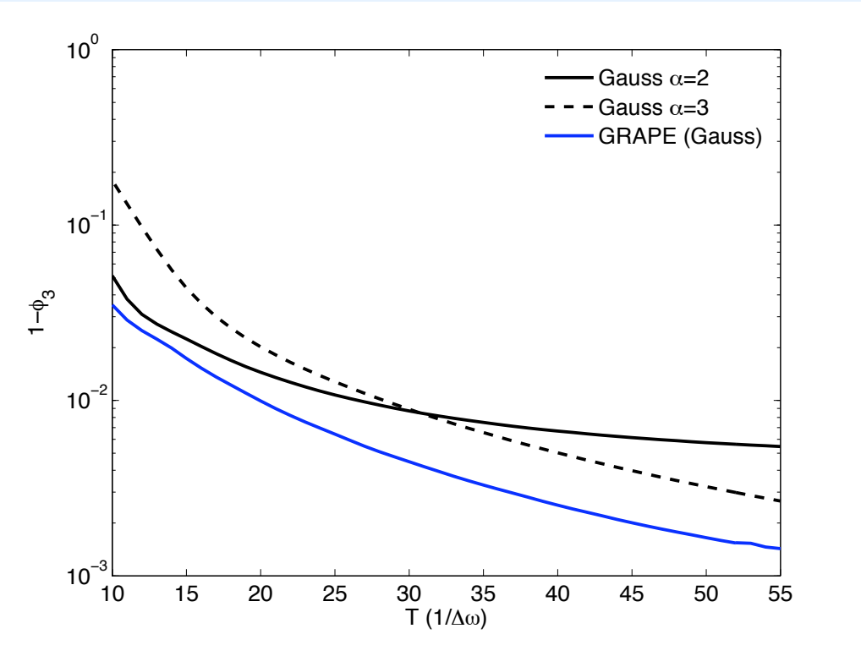
- Smoothly rising GRAPE pulses still outperform other shapes
- GRAPE helps to optimize within classes of pulse shapes (here Gaussians)
- Next step is to find high fidelity pulses intermediate between Gaussian and GRAPE pulses that are feasible for experiment



GRAPE pulse with smooth rise



Error of pulses versus total pulse time

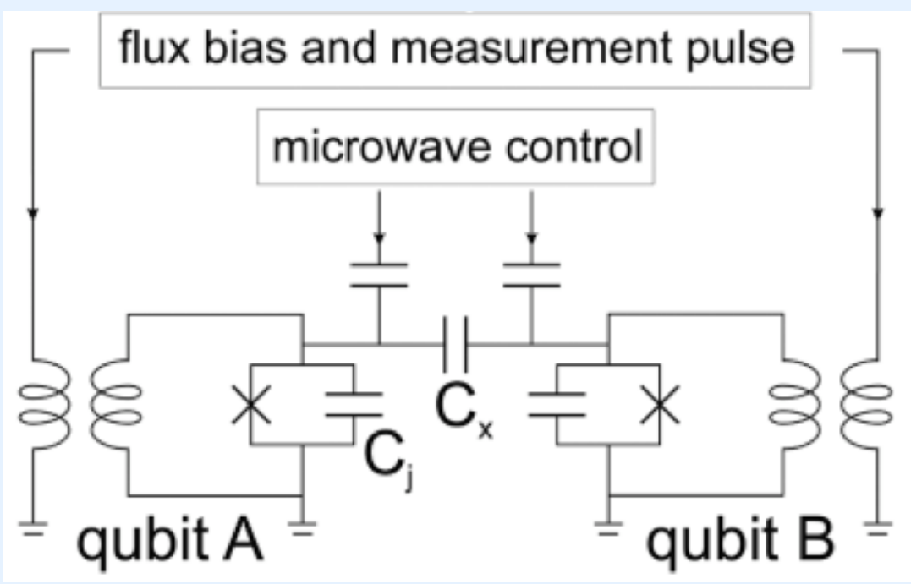


Optimized Gaussians, error versus total pulse time

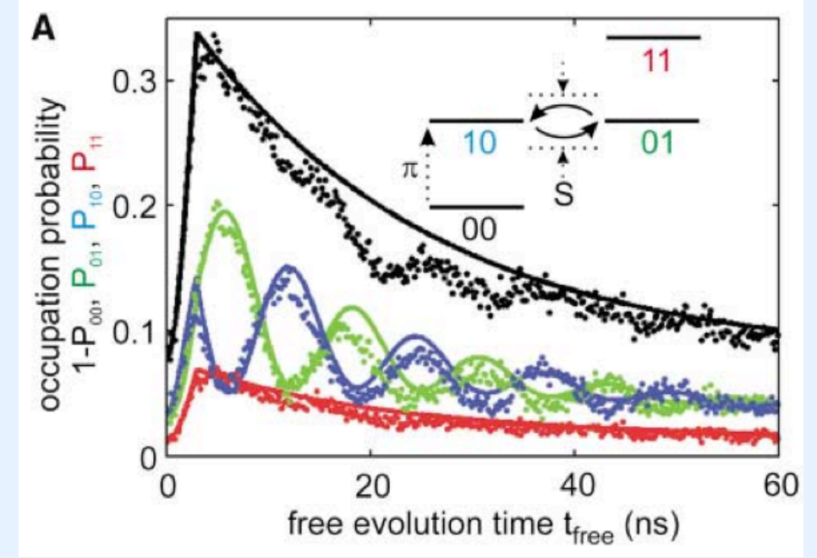
Coupled phase qubits

Two coupled three level systems. Resonant RWA Hamiltonian (two level notation for simplicity)

$$H = \lambda_1 X \otimes \mathbf{1} + \lambda_2 Z \otimes \mathbf{1} + \lambda_3 \mathbf{1} \otimes X + \lambda_4 \mathbf{1} \otimes Z + \frac{S}{4}(X \otimes X + Y \otimes Y)$$



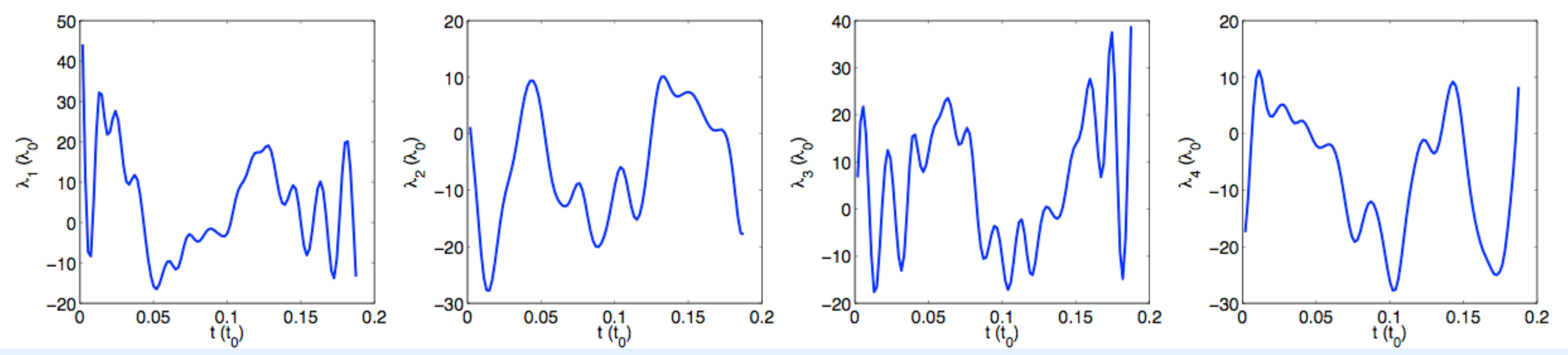
Circuit diagram for coupled phase qubits [4]



Experimental observation of coupling evolution [4]

Optimal two qubit gates

$$\phi = \frac{1}{16}|(U_F^\dagger U(T))_{55} + (U_F^\dagger U(T))_{66} + (U_F^\dagger U(T))_{88} + (U_F^\dagger U(T))_{99}|^2 \quad \mathrm{ISWAP}:\ U_{\mathrm{F}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



ISWAP pulse for coupled phase qubits

Conclusion and outlook

- In this setup, close non-qubit levels do not prevent us from implementing high fidelity qubit gates; GRAPE populates and depopulates these levels during pulse application
- Our single qubit pulses have fidelities of orders of magnitude higher than conventional shapes
- GRAPE finds two qubit gate pulses with fidelity above 0.9999

- Optimize our pulses with respect to pulse shaping capabilities of experimentalists
- Search for simpler and more time-optimal two qubit pulses
- GRAPE with decoherence

Acknowledgements

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References

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[4] R. McDermott et al., Science 307, 1299 (2005)